# Introduction to Nonlinear Control

## Stability, control design, and estimation

Christopher M. Kellett & Philipp Braun

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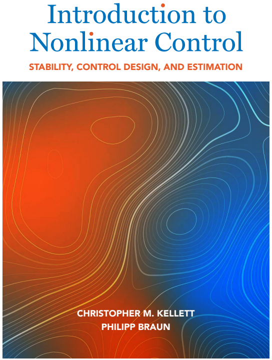

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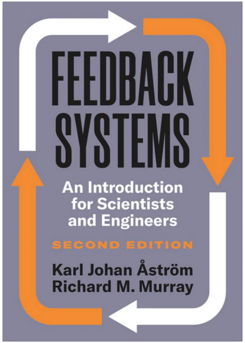
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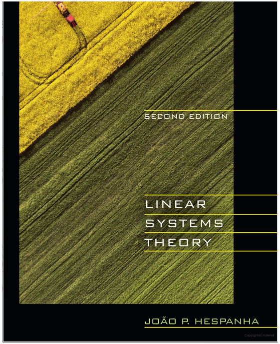
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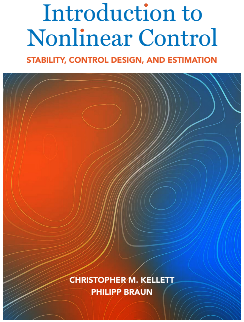
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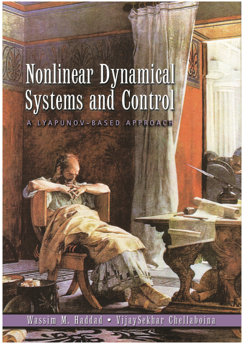
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# Dynamical Systems and the impact of Lyapunov methods

Dynamical Systems (with and without input)

$$\frac{d}{dt}x = \dot{x} = f(x), \qquad \frac{d}{dt}x = \dot{x} = f(x,u)$$

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  guaranteed? (Asymptotic stability)
- How to select a $u(x)$ such that convergence in (1) is guaranteed? (Stabilization)

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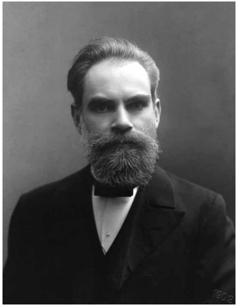
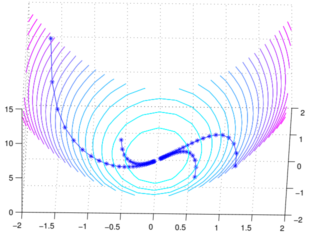
Lyapunov functions: Let $V : \mathbb{R}^n \to \mathbb{R}_{\geq 0}$ such that

$$\frac{d}{dt}V(x(t)) = \langle \nabla V(x(t)), f(x(t)) \rangle < 0 \quad \forall \ x(t) \neq x^*.$$

Then $x(t) \to x^*$ can be concluded.



Aleksandr Lyapunov
(1857–1918)



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