Introduction to Nonlinear Control

Stability, control design, and estimation

Christopher M. Kellett & Philipp Braun





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Part I: Dynamical Systems

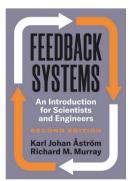
- 1. Nonlinear Systems -Fundamentals & Examples
- 2. Nonlinear Systems Stability Notions
- 3. Linear Systems and Linearization
- 4. Frequency Domain Analysis
- 5. Discrete Time Systems
- 6. Absolute Stability
- 7. Input-to-State Stability

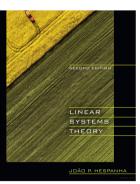
Part II: Controller Design

- 8. LMI Based Controller and Antiwindup Designs
- 9. Control Lyapunov Functions
- 10. Sliding Mode Control
- 11. Adaptive Control
- 12. Introduction to Differential Geometric Methods
- 13. Output Regulation
- 14. Optimal Control
- 15. Model Predictive Control

Part III: Observer Design & Estimation

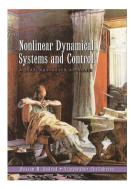
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Introduction to Nonlinear Control STABILITY. CONTROL DESIGN, AND ESTIMATION





Dynamical Systems and the impact of Lyapunov methods

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Lyapunov functions: Let $V : \mathbb{R}^n \to \mathbb{R}_{\geq 0}$ such that

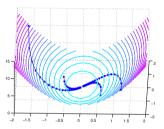
$$\frac{d}{dt}V(x(t)) = \langle \nabla V(x(t)), f(x(t)) \rangle$$

< 0 $\forall x(t) \neq x^*.$

Then $x(t) \rightarrow x^*$ can be concluded.



Aleksandr Lyapunov (1857–1918)



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