

Introduction to Nonlinear Control

Stability, control design, and estimation

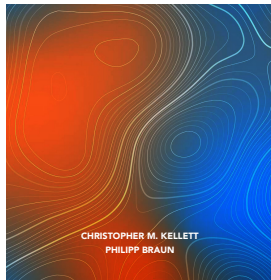
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Introduction to Nonlinear Control

STABILITY, CONTROL DESIGN, AND ESTIMATION

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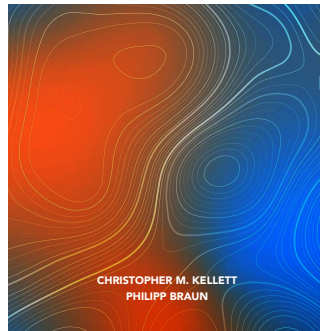
Part II: Controller Design

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Introduction to Nonlinear Control

STABILITY, CONTROL DESIGN, AND ESTIMATION



Introductory Examples: (Example 1)

Consider (nonlinear system)

$$\dot{x} = x^3 + u, \quad y = x.$$

Goal:

- Stabilize the state/output $y = x = 0$.

Solution:

- **Linear Controller Design:**

- ▶ Linearization about the origin:

$$\dot{x} = u$$

Natural stabilizing controller selection

$$u = -kx \quad (k > 0)$$

Nonlinear closed loop dynamics:

$$\dot{x} = x(x^2 - k), \quad x^e \in \{0, \pm\sqrt{k}\}.$$

- **Note that:**

- ▶ Origin is locally asymptotically stable.
- ▶ Region of attraction increases with k .

- **Nonlinear Controller Design:**

- ▶ Consider the nonlinear feedback

$$u = -x^3 + v, \quad v \text{ to be designed}$$

Closed-loop system

$$\dot{x} = v$$

Natural feedback selection

$$v = -kx \quad (k > 0)$$

Closed-loop system with globally asymptotically stable origin k :

$$\dot{x} = -kx$$

Overall feedback law:

$$u = -x^3 - kx$$

- **Note that:**

- ▶ Coordinate transformation in the input u leads to a linear system.
- ▶ Locally both control laws stabilize the origin, but the coordinate transformation provides a stronger result.

Introductory Examples: (Example 2)

Consider second-order system

$$\begin{aligned}\dot{x}_1 &= x_2 + x_1^2 \\ \dot{x}_2 &= -2x_1^3 - 2x_1x_2 + u \\ y &= x_1,\end{aligned}$$

Consider change of coordinates

$$\begin{aligned}z_1 &= x_1 \\ z_2 &= x_2 + x_1^2\end{aligned}$$

System in new coordinates:

$$\begin{aligned}\dot{z}_1 &= z_2 \\ \dot{z}_2 &= \dot{x}_2 + 2x_1\dot{x}_1 = u \\ y &= z_1,\end{aligned}$$

Note that: The system is linear in z !

Globally exponentially stabilizing feedback law:

$$\begin{aligned}u &= -k_1z_1 - k_2z_2 \\ &= -k_1x_1 - k_2(x_2 + x_1^2)\end{aligned}$$

for $k_1, k_2 > 0$.

(Can be easily verified by checking the eigenvalues of the closed-loop system.)

Note that:

- Coordinate transformation allows us to stabilize and analyze a linear system instead of a nonlinear system.
- $x \rightarrow 0$ is equivalent to $z \rightarrow 0$.
- For the input-output behavior it is not important if the dynamics are written in terms of x or z .

Introductory Examples: (Example 3)

Consider third-order nonlinear system:

$$\begin{aligned}\dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3^3 + u \\ \dot{x}_3 &= x_1 + x_3^3\end{aligned}\quad y = x_1,$$

Consider change of coordinates:

$$\begin{aligned}z_1 &= x_3 \\ z_2 &= x_1 + x_3^3 \\ z_3 &= x_2 + 3x_1x_3^2 + 3x_3^5.\end{aligned}$$

and initial feedback (with v to be designed):

$$u = -x_3^3 - 3x_2x_3^2 - 6x_1^2x_3 - 21x_1x_3^4 - 15x_3^7 + v,$$

Leads to linear states (but a nonlinear output):

$$\begin{aligned}\dot{z}_1 &= z_2 \\ \dot{z}_2 &= z_3 \\ \dot{z}_3 &= v\end{aligned}\quad y = z_2 - z_1^3.$$

The feedback law

$$u = -x_3^3 + v$$

Leads to linear input-output relationship from v to y :

$$\begin{aligned}\dot{x}_1 &= x_2 \\ \dot{x}_2 &= v \\ \dot{x}_3 &= x_1 + x_3^3\end{aligned}\quad y = x_1,$$

Here,

- the “internal” x_3 dynamics, which are not visible through the output, are nonlinear.
- we are able to partially linearize the dynamics
- For the linear dynamics, v can be defined such that the origin $z = 0$ is asymptotically stable (i.e., y converges to zero).
- For the nonlinear dynamics a controller guaranteeing $y(t) \rightarrow 0$ for $t \rightarrow \infty$ can be defined using pole placement. But is the origin asymptotically stable?

In this chapter we will discuss . . .

Feedback linearization

- Input-to-state linearization
- Input-to-output linearization

Relies on properties such as

- relative degree
- zero dynamics

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- coordinate transformation of the input

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Relies on concepts such as

- coordinate transformation of the state
- coordinate transformation of the input
- (repeated) Lie derivatives ($\lambda : \mathbb{R}^n \rightarrow \mathbb{R}^m$,
 $f : \mathbb{R}^n \rightarrow \mathbb{R}^n$)

$$L_f \lambda(x) = \frac{\partial \lambda}{\partial x}(x) \cdot f(x)$$

$$L_f^k \lambda(x) = \frac{\partial}{\partial x} \left(L_f^{k-1} \lambda(x) \right) f(x), \quad L_f^0 \lambda(x) = \lambda(x)$$

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Feedback linearization

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- Input-to-output linearization

Relies on properties such as

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- zero dynamics

This also allows us to talk about

- nonlinear controllability (accessibility)

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Introduction to Nonlinear Control: Stability, control design, and estimation

Part I: Dynamical Systems

1. Nonlinear Systems - Fundamentals & Examples
2. Nonlinear Systems - Stability Notions
3. Linear Systems and Linearization
4. Frequency Domain Analysis
5. Discrete Time Systems
6. Absolute Stability
7. Input-to-State Stability

Part II: Controller Design

8. LMI Based Controller and Antiwindup Designs
9. Control Lyapunov Functions
10. Sliding Mode Control
11. Adaptive Control
12. Introduction to Differential Geometric Methods
13. Output Regulation
14. Optimal Control
15. Model Predictive Control

Part III: Observer Design & Estimation

16. Observer Design for Linear Systems
17. Extended & Unscented Kalman Filter & Moving Horizon Estimation
18. Observer Design for Nonlinear Systems