Introduction to Nonlinear Control

Stability, control design, and estimation

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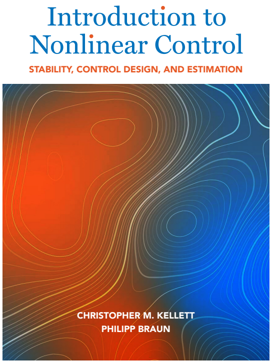
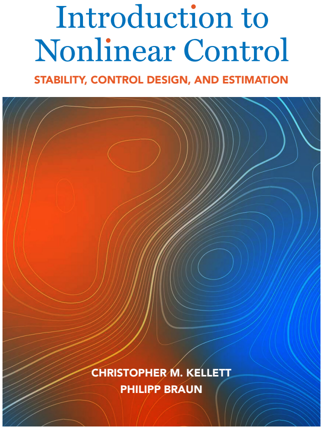
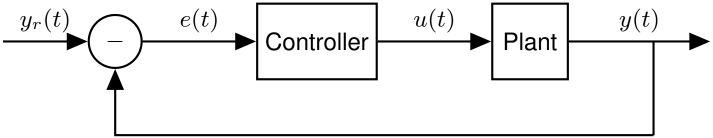


Part II: Controller Design



Output Regulation



Consider perturbed linear system:

$$\dot{x} = Ax + Bu + E_d d$$
$$y = Cx + Du + F_d d,$$

with disturbance d

Goal:

- asymptotically track a reference signal

$$y(t) \to y_r(t)$$

(regardless of the disturbance)

Tracking error:

$$e(t) = y(t) - y_r(t).$$

i.e., $\quad e = Cx + Du + F_d d - y_r.$

Assumption: Reference and disturbance satisfy

$$\dot{y}_r = A_r y_r \qquad \text{and} \qquad \dot{d} = A_d d,$$

Additional Assumptions/Definitions:

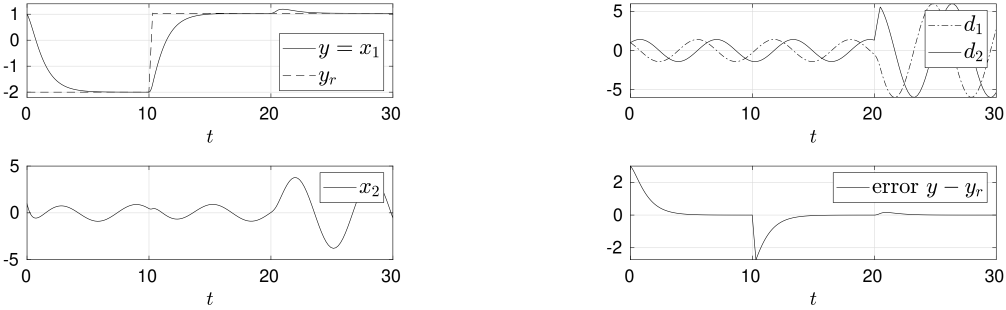
- Exosystem:

$$w = \begin{bmatrix} y_r \\ d \end{bmatrix}, \quad \dot{w} = A_1 w, \quad A_1 = \begin{bmatrix} A_r & 0 \\ 0 & A_d \end{bmatrix}.$$

- Overall system dynamics:

$$\begin{aligned} \dot{x} &= Ax + Bu + [0 \;\; E_d]w = Ax + Bu + Ew, \\ \dot{w} &= A_1 w \\ e &= Cx + Du + [-I \;\; F_d]w = Cx + Du + Fw \end{aligned}$$

Example



- Tracking of a piecewise constant reference signal
- The frequency of the disturbance d changes after 20 seconds
- Only convergence of the output but not the state is guaranteed

Output Regulation: Problem Formulation and Controller design

(Linear) Output regulation problem:

- For

$$\dot{x} = Ax + Bu + [0 \;\; E_d]w = Ax + Bu + Ew$$
$$\dot{w} = A_1 w$$
$$e = Cx + Du + [-I \;\; F_d]w = Cx + Du + Fw$$

define

$$u = K_x + K_w w$$

such that

- $\lim_{t\to\infty} e(t) = 0$
- $\lim_{t\to\infty} x(t) = 0$ in the case $w \equiv 0$.

It holds that

$$\dot{x}_c = (A + BK_x)x_c + (E + BK_w)w = A_c x_c + B_c w$$
$$\dot{w} = A_1 w$$
$$e = (C + DK_x)x_c + (F + DK_w)w = C_c x_c + D_c w$$

Task: Find K_x and K_w such that

- A_c is Hurwitz and
- $\lim_{t\to\infty} e(t) = \lim_{t\to\infty} (C_c x_c(t) + D_c w(t)) = 0$

Lemma

Assume A_c is Hurwitz and A_1 has no eigenvalues with negative real parts. Then $\lim_{t\to\infty} e(t)$ if and only if there exists a unique matrix X_c satisfying

$$X_c A_1 = A_c X_c + B_c,$$
$$0 = C_c X_c + D_c.$$

Controller design:

1. Select K_x so that $A_c = A + BK_x$ is Hurwitz.
2. Solve (unknowns: X_c and K_w)

$$X_c A_1 = (A + BK_x)\, X_c + BK_w + E$$
$$0 = (C + DK_x)\, X_c + DK_w + F$$

Output Regulation: Motivation and Problem Formulation

Controller design:

1. Select K_x so that $A_c = A + BK_x$ is Hurwitz.
2. Solve (unknowns: X_c and K_w)

$$\begin{aligned} X_c A_1 &= (A + BK_x) X_c + BK_w + E \\ 0 &= (C + DK_x) X_c + DK_w + F \end{aligned}$$

Drawback: K_x and K_w are designed sequentially
Consider coordinate transformation:

$$\begin{bmatrix} X \\ U \end{bmatrix} = \begin{bmatrix} I & 0 \\ K_x & I \end{bmatrix} \begin{bmatrix} X_c \\ K_w \end{bmatrix}$$

leading to (*regulator equations*)

$$\begin{aligned} XA_1 &= AX + BU + E \\ 0 &= CX + DU + F \end{aligned}$$

- linear in unknowns X, U
- $K_w = U - K_x X$ for all K_x

Theorem (Regulator equations)

The regulator equations are solvable for any matrices E and F if and only if for all eigenvalues λ of A_1, it holds that

$$\operatorname{rank}\left(\begin{bmatrix} A - \lambda I & B \\ C & D \end{bmatrix} \right) = n + p$$

(where the matrix has dimension $(n+p) \times (n+p)$)

Extension 1: Uncertain linear systems

$$\begin{aligned} \dot{x} &= A(\delta)x + B(\delta)u + E(\delta)u \\ e &= C(\delta)x + D(\delta)u + F(\delta)u \end{aligned}$$

Extension 2: Nonlinear Output Regulation:

$$\begin{aligned} \dot{x} &= F(x, u, w) \\ \dot{w} &= a_1(w) \\ e &= H(x, u, w) \end{aligned}$$

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